

parameters. (ξ, ζ) .

$$\xi \in X^*(T) = X_*(T') \subseteq X_*(G')(K).$$

$$\zeta \in T'(K) \subseteq G'(K) \quad \text{s.t.} \quad \zeta \in T'_{\xi, \text{norm}}$$

\downarrow

$B_{\xi, \zeta}$ unitary Banach space rep of G .

\rightarrow not expected to be irred.

Conj $B_{\xi, \zeta} \neq 0$.

$$T'_{\xi, \text{norm}} := \text{val}^{-1}(V_{\xi}^{\text{norm}})$$

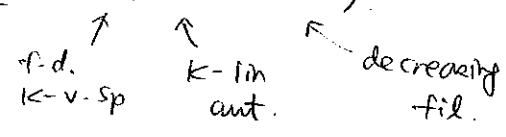
$$V_{\xi}^{\text{norm}} = \{z \in V_{\mathbb{R}} : z^{\text{dom}} \leq \eta_{\mathbb{Q}_p} + \xi_{\mathbb{Q}_p}\}$$

$$V_{\xi} := \{z \in V_{\mathbb{R}} : (z + \eta_{\mathbb{Q}_p})^{\text{dom}} \leq \eta_{\mathbb{Q}_p} + \xi_{\mathbb{Q}_p}\}$$

no semilinear b/c base is \mathbb{Q}_p .

$\text{FIC}_K :=$ categ. of filtered K -isocrystals

$$\underline{D} = (D, (\varphi, \text{Fil} \circ D))$$



$\text{FIC}_K^{\text{adm}} \subseteq \text{FIC}_K$ full subcat. of (weakly) adm obj's.

Faltings/Totaro: $\text{FIC}_K^{\text{adm}}$ is a neutral Tannakian categ
 \uparrow implies (Jay says).

Colmez/Fontaine: $\text{FIC}_K^{\text{adm}} = \text{Rep}_K^{\text{crys}}(\text{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p))$

A

- $G = \text{GL}_{d+1}(\mathbb{Q}_p)$
- U_1
- P lower ∇ Borel.
- U_1
- T diag. mat.

$$U_0 := \text{GL}_{d+1}(\mathbb{Z}_p).$$

i^{th} spot.

$$T/T_0 = \Lambda \ni \lambda_i := \left(\begin{matrix} 1 & & & \\ & \downarrow & & \\ & p_1 & & \\ & & \ddots & \\ & & & 1 \end{matrix} \right) T_0.$$

$$V_{\mathbb{R}} = \text{Hom}(\Lambda, \mathbb{R}) \cong \mathbb{R}^{d+1}$$

$$z \mapsto (z(\lambda_1), \dots, z(\lambda_{d+1}))$$

$$X^*(T) \otimes \mathbb{R} \cong V_{\mathbb{R}} \cong \mathbb{R}^{d+1}$$

$$\zeta = \left[\begin{pmatrix} \zeta_1 \\ \vdots \\ \zeta_{d+1} \end{pmatrix} \mapsto \Pi \zeta = \begin{pmatrix} a_1 \\ \vdots \\ a_{d+1} \end{pmatrix} \right] \mapsto (a_1, \dots, a_{d+1})$$

dominant $a_1 \leq \dots \leq a_{d+1}$

$$\eta_{\mathbb{Q}_p} \cong \frac{1}{2}(-d, -(d-2), \dots, d-2, d)$$

note: $\tilde{\eta}_{\mathbb{Q}_p} = (0, 1, \dots, d) = \eta_{\mathbb{Q}_p} + \frac{1}{2}(d, \dots, d)$

W-inv.

adding this to def of $V_{\mathbb{Z}}$

$$V_{\mathbb{Z}} = \left\{ z : (z + \tilde{\eta}_{\mathbb{Q}_p})^{\text{dom}} \leq \tilde{\eta}_{\mathbb{Q}_p} + \zeta_{\mathbb{Q}_p} \right\}$$

now the formula involves integ. wts only.

use on T' the coord.

$$T'(K) = \text{Hom}(\Lambda, K^{\times}) \rightarrow (K^{\times})^{d+1}$$

$$\zeta \mapsto (\zeta(\lambda_1), p^{\zeta}(\lambda_2), \dots, p^d \zeta(\lambda_{d+1}))$$

Fact Under these coord. $T'_{\mathbb{Z}} = \text{val}^{-1}(V_{\mathbb{Z}})$ corresp. to

$$\left(w_p(\zeta_1), \dots, w_p(\zeta_{d+1}) \right)^{\text{dom}} \leq (a_1, a_2+1, \dots, a_{d+1}+d)$$

Hodge - Newton polygon!

moreover: ① $(\cdot)^{\text{dom}}$ means rearranging in an increasing order.

$$\textcircled{2} (z_1, \dots, z_{d+1}) \leq (z'_1, \dots, z'_{d+1}) \text{ if}$$

$$z_{d+1} \leq z'_{d+1}, z_1 + z_{d+1} \leq z'_1 + z'_{d+1}, \dots, z_2 + \dots + z_{d+1} \leq z'_2 + \dots + z'_{d+1}$$

$$z_1 + \dots + z_{d+1} = z'_1 + \dots + z'_{d+1}$$

D in FIC_K of dim $d+1$.

(jump index)

$$\rightsquigarrow * \varphi \in \text{GL}_{d+1}(K) = G(K)$$

where filtr.

$$* \text{ Fil } D \text{ has a type, } \left[\text{type}(D) \right]$$

jumps

which could be viewed as a dominant elt in $X^*(T)$

Thm For $\xi \in X^*(T)$ dominant and $\zeta \in T'(K)$, TFAE.

- $\zeta \in T'_\xi$.
- \exists a \underline{D} in FIC_K^{adm} s.t. $\varphi^{\xi} = \zeta$ and $type(D) = \xi$.

(admissibility = condition on all sub isocrs.)

using the Fontaine ftr

our param. $(\xi, \zeta) \mapsto$ family of crys. Gal rep.

(HT wt mult = 1 restriction...)

$B_{\xi, \zeta} \longmapsto$ family of ~~quots~~ all top. imed quots.

B G general (split)

$REP_K(G')$:= categ of K -rat'l rep of G'
neutral Tannakian categ.

Let's look at a general pair

$$(v, b) \in X_*(G')(K) \times G'(K)$$

we have ftr

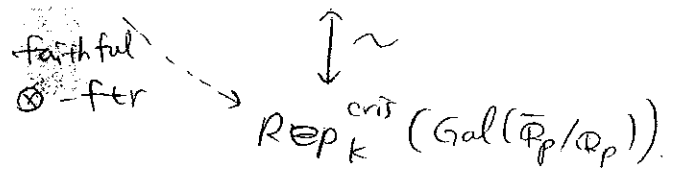
$$I_{(v,b)} : REP_K(G') \rightarrow FIC_K$$

$(\rho, E) \mapsto (E, \rho(b))$, wt fil of cochar $\rho \circ v$.

Def (v, b) is called adm if $I_{(v,b)}$ has image in FIC_K^{adm} .

(v, b) adm.

$$\Rightarrow REP_K(G') \xrightarrow{I_{(v,b)}} FIC_K^{adm}$$



~~(v, b) adm~~

~~REP_K(G) → FIC_K^{adm}~~

Tanaka formalism.

⇒ $\gamma(v, b) : \text{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p) \rightarrow G'(\mathbb{K})$

det'd up to conj. on the target.

harmless

b/c this is "Langlands param."

can take K^{ur} .

Stemberg thm. sum of coh. fib for isom / K^{ur} ?

Problems 1) fertility forces us to use $T'_{\xi, \text{norm}}$.
→ need $p^{\frac{1}{2}} \in K$.

2) $\eta_{\mathbb{Q}_p}$ not integral.

Thm. Let $\xi \in X_*(T')$, dominant & $\zeta \in T'(K)$.
assume $\eta_{\mathbb{Q}_p}$ is integral. TFAE

(i) $\zeta \in T'_{\xi, \text{norm}}$.

(ii) \exists an adm pair (v, b) s.t.

$v \in G'(K)$ -orbit of $\zeta \eta_{\mathbb{Q}_p}$ and $b^{ss} = \zeta$.
conj. action.

what if $\eta_{\mathbb{Q}_p}$ is not integral?

requires \exists of $\sqrt{\text{of cyclo char}}$

Thm Everything remains the same

* if we work with filtered isocrs. whose fil. are indexed by $\frac{1}{2}\mathbb{Z}$.

* and if we replace $\text{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p)$ by its unique ^{non triv} central extn (classified by $H^2(G_{\mathbb{Q}_p}, \{\pm 1\}) = \{\pm 1\}$)

$(-1) \{ \pm 1 \} \rightarrow \text{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p)_{(2)} \rightarrow \text{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p) \rightarrow 1$

provided K is big enough.

⊛

but cat of is too big?

⊗ means, this is surj, so $\exists \sqrt{\quad}$ of ϵ .

$$\begin{array}{ccccc}
 G_{\mathbb{Q}_p(2)} & \longrightarrow & G_{\mathbb{Q}_p} & \longrightarrow & 0 \\
 \downarrow \sqrt{\text{of } \epsilon} & & \downarrow \epsilon \text{ - cydo char} & & \\
 E_{(2)} & & K^\times & \longrightarrow & 1 \\
 \downarrow 2 & & & & \\
 K^\times & \longrightarrow & K^\times & \longrightarrow & 1
 \end{array}$$

adm.
FIC_{K,2}

U

{ \mathcal{D} -subcat. gen'd by
FIC_K^{adm} and $(\sqrt{\epsilon})$ }

twist -1 compo by this.

before crys. rep by requiring

this is crys.

END

(pf) Tannakian formalism

To

* Any Tannak. cat., with a choice of fiber ftr (to the cat. of v. sp.

(roughly what I heard)

we can assoc. a gp. (pro-alg. gp) whose rep. categ is

equiv to the Tan. cat. There may be many such gps.

$$G \rightarrow G' \text{ induces } \text{Rep}(G) \leftarrow \text{Rep}(G')$$

$$T_1, T_2 = \text{Tann. cat.} \quad T_1 \xrightarrow{f} T_2$$

any f s.t. diag. comm.

$$\begin{array}{ccc} (f_1) & \searrow & \swarrow (f_2) \\ & \text{Vec} & \end{array}$$

fiber ftr. Vec

$$\Downarrow 1-1$$

$$\text{Aut}(f_1) \leftarrow \text{Aut}(f_2)$$

aut. gp of fib ftr.

In our case,

$$\begin{array}{ccc}
 \text{Rep}(G') & \longrightarrow & \text{Rep}(\text{Gal}) \\
 \text{forget} \downarrow & \searrow (f_0) \circlearrowleft & \downarrow \text{forget} \\
 \text{Vec} & \text{Composite Vec} & \text{Vec}
 \end{array}$$

$$\begin{array}{ccc}
 \text{Aut}(\text{forget})(K^{ur}) & \cong & \text{Aut}(f_0)(K^{ur}) \\
 \parallel & & \uparrow \\
 G'(K^{ur}) & & \text{contains Gal.} \\
 & \nearrow \text{choice.} &
 \end{array}$$

existence can be seen by Steinberg thm.

(up to inner aut).

$$H^1(K^{ur}, G') = 0$$