Sample Questions from Past Qualifying Exams

This list may give the impression that the exams consist of a series of questions fired at the student one after another. In fact most exams have more the character of a conversation with considerable give and take. Hence this list cannot be expected to indicate accurately the difficulties involved.

The list indicates the professor associated to each question where available. Some have been in the MGSA files for a while, and this information has been lost (if it was ever there).

The listing by section is approximate, since some questions may fit under more than one heading.

Algebraic Geometry

- What are the involutions of an elliptic curve over $\mathbb{C}$? [**McMullen**]
- What quotient arises from this involution? [**McMullen**]
- What are the fixed points of this involution? [**McMullen**]
- So how can you show this quotient is $\hat{\mathbb{C}}$? [**McMullen**]
- Let's talk about Riemman-Hurwitz. Given a nonconstant map between curves over k, is there an associated map on differentials? A resulting exact sequence? [**Ogus**]
- Is the exact sequence short exact in this case? [**Ogus**]
- Now can you prove the weak version of Riemann-Hurwitz? [**Ogus**]
- Calculate $\mathrm{Pic}(k[t^2,t^3])$. $k[t^2,t^3] \subset k[t]$. [**Ogus**]
- Find an example of a projective curve which is not rational.
- Is $\mathbb{P}^1 \times \mathbb{P}^1$ a projective variety? Prove it.
- Find the explicit equation of the image of $\mathbb{P}^1 \times \mathbb{P}^1$ under the Segre embedding

$$\psi(\mathbb{P}^1 \times \mathbb{P}^1) \subseteq \mathbb{P}^3$$

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- If the field is $\mathbb{C}$, the embedding in $\mathbb{P}^3$ is the 4-dimensional manifold. Compute the intersection form.
- How do you use Hurwitz's formula to calculate the genus of a given curve? [**Coleman**]
- What can you say about curves over perfect fields? [**Coleman**]
- Show that a hypersurface defined by an equation of degree d has degree d. [**Sturmfels**]
- What does the degree (leading term of $P_X(r)$) have to do with line bundles on $\mathbb{P}^1$ (namely, $\mathcal{O}(3)$)? [**Ogus**]
- What does the constant term of $P_X(r)$ represent?
- Let X be the twisted cubic in $\mathbb{A}^3$. Is X an intersection (set-theoretically) of two surfaces in $\mathbb{P}^3$? [**Ogus**]

- Is the twisted cubic in $\mathbb{A}^3$ the intersection (scheme-theoretically) of two surfaces in $\mathbb{P}^3$? [**Ogus**]
- What can you say about curves $Y \leq \mathbb{A}^3$ and $Y \equiv \mathbb{A}^1$: are they (set,scheme)-theoreticallly intersections of two surfaces? [**Ogus**, later recanted]
- Define separated morphism. [**Ogus**]
- Give an example of a non-separated morphism. [**Poonen**]
- Do you know what quasi-separated means? [**Ogus**]
- Name a good property of separated morphisms. [**Ogus**]
  - What would be the analogue for quasiseparated in place of separated? [**Ogus**]
- What can you say about separated schemes? [**Ogus**]
  - Say $g, h: Z \to X$, with Z and X schemes over Y, via $f: X \to Y$, and g and h agreeing on an open dense subset of Z. What can be said if f is separated? If Z is reduced? [**Ogus**]
  - Give examples where Z is not reduced or f not separated and $g \neq h$. [**Ogus**]
- Define differentials. [**Ogus**]
- Are differentials quasicoherent? [**Ogus**]
- What does the going up theorem mean in algebraic geometry? [**Hartshorne**]
- What can you say about the dimension of the image of a map from $\mathbb{P}^n$ to $\mathbb{P}^m$? [**Hartshorne**]
- What is the genus of a curve? [**Hartshorne**]
- Does the genus of a curve depend on the embedding? [**Hartshorne**]
- When is a canonical divisor very ample? [**Wodzicki**]
- State Riemann-Roch. [**Wodzicki**]
- Compute the dimension of the space of holomorphic differentials on a Riemann surface of genus g. [**Wodzicki**]
- State Abel's theorm. [**Wodzicki**]
- What is the significance of the Jacobian? What kind of map is the Abel-Jacobi map? What is it in the case of genus 1? [**Wodzicki**]
- What is the connection between H^1 and line bundles? [**Wodzicki**]
- What is a scheme? [**Ogus**]
- How can you tell if a scheme is affine? [**Ogus**]
- Can you weaken the Noetherian hypothesis in Serre's criterion for affineness? [**Ogus**]
- Prove that if X is a Noetherian scheme such that $H^1(X, I) = 0$ for all coherent sheaves of ideals I then X is affine. [**Ogus**]
- Can you give an example where the theorem is false if we drop the quasi-compactness assumption? [**Ogus**]
- What can you say about curves of genus 0? [**Ogus**]

- Prove that such a curve is always isomorphic to $\mathbb{P}^1$ or can be embedded as a quadric in $\mathbb{P}^2$. [**Ogus**]
- If the base field is a finite field, can the latter case occur? [**Ogus**]
- Calculate $H^0(\mathbb{P}^1, \Omega^1)$. [**Poonen**]
- If $f(x,y)$ and $g(x,y)$ are two polynomials such that the curves they define have infinitely many points in common, is it true that they have a common factor?
- Give two criteria for a curve to be nonsingular (over an algebraically closed field). [**Ogus**]
- What is a normal domain? How is this related to regular local rings? [**Ogus**]
- Find the singularities—if any—of the curve in $\mathbb{P}^2$ defined by the equation $X^3 + Y^3 + Z^3 = 3CXYZ$. [**Ogus**]
- Describe Weil divisors and Cartier divisors on curves. [**Ogus**]
- How do you get a Weil divisor from an element $f \in K^*$, in the canonical isomorphism? [**Ogus**]
- What is the degree of a divisor? [**Ogus**]
- Does there exist a variety V with $\mathrm{Pic}(V) = \mathbb{Z}/3\mathbb{Z}$? [**Poonen**]
- Does there exist a projective variety V with $\mathrm{Pic}(V) = \mathbb{Z}/3\mathbb{Z}$? [**Poonen**]
- Is the complement of a hypersurface in $\mathbb{P}^2$ affine? [**Poonen**]
- Define the geometric genus. [**Poonen**]
- What might be the geometric genus of a singular curve? [**Poonen**]
- Find the arithmetic genus of $y^3 = x^2 z$. [**Frenkel**]