Sample Questions from Past Qualifying Exams

This list may give the impression that the exams consist of a series of questions fired at the student one after another. In fact most exams have more the character of a conversation with considerable give and take. Hence this list cannot be expected to indicate accurately the difficulties involved.

The list indicates the professor associated to each question where available. Some have been in the MGSA files for a while, and this information has been lost (if it was ever there).

The listing by section is approximate, since some questions may fit under more than one heading.

Basic Group Theory

- What can you say about finitely generated abelian groups?
- How many such groups are there of order 35, 27?
- What are the subgroups of these groups?
- Classify all abelian groups of 27 elements and prove that they are pairwise non-isomorphic [**Bergman**]
- Give an example of a non-abelian group of order 27. [**Bergman**]
- What can you say about groups of order pq?
- Show that any p-subgroup is contained in a p-Sylow subgroup.
- Show that any two p-Sylow subgroups are conjugate.
- Is the converse of Lagrange's theorem true?
- Give examples of a finite non-abelian group which cannot be mapped homomorphically onto a group of smaller order.
- Are the 2x2 matrices of determinant 1 simple?
- Can A_4 be mapped homomorphically onto $\mathbb{Z}/2\mathbb{Z}$?
- Is the normalizer of a subgroup a normal subgroup?
- Show that G/H, where H is the commutator subgroup, is abelian.
- Show that $\mathbb{R}/\mathbb{Z}$ is isomorphic to the rotations about the origin of the complex plane.
- Let G be the group of real numbers mod 1. What can be said about the subgroup generated by a single irrational number $r \in [0,1]$?
- Is it true that for any finite group G there is a polynomial over $\mathbb{Q}$ such that G is the Galois group of the splitting field over $\mathbb{Q}$ of this polynomial?
- Let G be a group of order 99. Does there exist a normal subgroup of order 11 in G?
- Show that in a group of order 36, there is a nontrivial normal subgroup which has order either 3 or 9.
- Let H be a subgroup of G such that $[G\colon H]$ is the smalles prime dividing the order of G. Show that H is normal.

- Show that any group of order 12 must have a normal subgroup.
- What is the order of the center of a nonabelian group of order p^3, for a prime p?
- Let G be a group of order p^r for some prime p. Show that G has nontrivial center.
- Show that any group of order p^r, p a prime, is solvable.
- Show that if p divides the order of an abelian group, p a prime, then there is a subgroup of order p.
- If G is a cyclic group, and r divides the order of G, how many subgroups of order r does G have?
- Suppose that every element of a finite gropu G has order a power of the same prime. Is it true that the order of G is a power of this prime?
- Let S_p be a Sylow p-subgroup of G. Prove that $N(N(S_p)) = N(S_p)$.
- Show that if $|G| < 60$ and $|G|$ is not a prime, then G is not simple.
- If every subgroup of a group is normal, can you conclude that the group is abelian?
- What are the Sylow Theorems?
- What is a free group?
- What can you say about the subgroups of a group of order 30? [**Stallings**]
- Write down a composition series for S_4. [**Bergman**]
  - Which groups in this series are normal? [**Bergman**]
- Describe the 3-Sylow subgroups of symmetric groups. [**Rhodes**]
- Give an outline of the proof of the Jordan-Dickson Theorem. [**Bergman**]
- Describe using a picture the Sylow 3-subgroups of $S_3, S_4, \ldots, S_9$.
- Describe the 2-Sylow subgroups of S_4, S_5, and S_6. [**Lam**]
- Prove: If G is a group generated by 2 involutions, then G is a dihedral group. [**Lam**]
- State and prove the Core Theorem. [**Bergman**]
- How many (essentially different) 6 beaded necklaces using 2 colors are there? [**Lam**]
- Talk about multiple transitivity; what does it mean for a G-set to be k-transitive? Sharply k-transitive? [**Bergman**]
- Prove that for a sharply 2-transitive G-set X, the size of the Frobenius kernel is equal to the size of X. [**Bergman**]
- What groups are sharply k-transitive for large values of k? [**Bergman**]