Sample Questions from Past Qualifying Exams

This list may give the impression that the exams consist of a series of questions fired at the student one after another. In fact most exams have more the character of a conversation with considerable give and take. Hence this list cannot be expected to indicate accurately the difficulties involved.

The list indicates the professor associated to each question where available. Some have been in the MGSA files for a while, and this information has been lost (if it was ever there).

The listing by section is approximate, since some questions may fit under more than one heading.

Measure Theory and Real Analysis

- Can you think of any interesting functors in Measure Theory? [**Rieffel**]
- Make $L_p(-)$ into a functor. [**Rieffel**]
- Make "measurable sets" into a functor. [**Rieffel**]
- Quote Caratheodory's Theorem. [**Rieffel**]
- Consider the unit interval. What can you say about the integral of a non-negative function on the unit square? [**Arveson**]
- What is the precise statement of the Fubini theorem? [**Arveson**]
- Consider the following definition for f a measurable function:
  (1) $f^{-1}(A)$ is measurable for all intervals A; and
  (2) $f^{-1}(A)$ is measurable for all Borel sets A.
  Are they the same? Why? [**Arveson**]
- If A is a subset of the real plane such that the intersection of A with any horizontal line is countable, what can you say about the Lebesgue measure of A? [**Arveson**]
- What if the intersection of A with every line of slope one is countable? [**Arveson**]
- Why is the map $(x, y) \mapsto (x, y - x)$ measure preserving? Is the map $(x, y) \mapsto (x, y - 3x)$? $(x, y) \mapsto (x, 3y - 2x)$? [**Arveson**]
- How do you know if a map is measure preserving? [**Arveson**]
- What does Alaoglu's theorem say? What is the basic idea of the proof of Alaoglu's theorem? [**Arveson**]
- Name somre results which use the Baire Category Theorem. [**Arveson**]
- What is the definition of the weak-* topology? [**Arveson**]