Sample Questions from Past Qualifying Exams

This list may give the impression that the exams consist of a series of questions fired at the student one after another. In fact most exams have more the character of a conversation with considerable give and take. Hence this list cannot be expected to indicate accurately the difficulties involved.

The list indicates the professor associated to each question where available. Some have been in the MGSA files for a while, and this information has been lost (if it was ever there).

The listing by section is approximate, since some questions may fit under more than one heading.

Representation Theory, Lie Groups, Lie Algebras

- Prove Engel's theorem. [**Serganova**]
- Prove Lie's theorem. What would happen if the hypothesis was not that g is solvable but that $g \neq [g, g]$? [**Serganova**]
- Why is $g = [g, g]$ for g semisimple? [**Weinstein**]
- What is the exponential map, and what is it good for? [**Serganova**]
- Classify the real connected abelian Lie groups. [**Serganova**]
- Prove that a Lie group homomorphism $\phi : H \to G$ for H connected is determined by the derivative at the identity. [**Serganova**]
- Give an example of a Lie group G where the exponential map is not surjective. [**Weinstein**]
- Given the standard representation of $sl_n(\mathbb{C})$ identify the simple roots and explain the correlation between the height of the root and the corresponding "location" in the matrix. [**Frenkel**]
- Decompose $Sym_n(V) \otimes Sym_m(V)$ where V is the 2-dimensional irreducible representation of $sl_2(\mathbb{C})$. [**Frenkel**]
- Do the calculation above using a character formula. [**Reshetikhin**]
- State and explain the Harish-Chandra isomorphism. [**Wodzicki**]
- Explain how to write down the Weyl group of SL_n using generators and relations. [**Frenkel**]
- What is a Verma module? [**Reshetikhin**]
- When is a Verma module finite-dimensional? [**Wodzicki**]