Sample Questions from Past Qualifying Exams

This list may give the impression that the exams consist of a series of questions fired at the student one after another. In fact most exams have more the character of a conversation with considerable give and take. Hence this list cannot be expected to indicate accurately the difficulties involved.

The list indicates the professor associated to each question where available. Some have been in the MGSA files for a while, and this information has been lost (if it was ever there).

The listing by section is approximate, since some questions may fit under more than one heading.

Universal Algebra

- Take a non-trivial ultraproduct of the field $\mathbb{Z}/p\mathbb{Z}$ over all primes p. Is it a field? What is its characteristic? Is it algebraically closed? [**Bergman**]
- Make "generators and relations" into a functor. What is its left adjoint? [**Bergman**]
- What is a Galois connection? Give some examples. [**Rieffel**]
  Must a Galois connection arise from a relation? [**Bergman**]
- Let R be a relation on X which is reflexive and symmetric, X a set. Use R to define a Galois connection between $\wp(X)$ and $\wp(X)$. Show that the following are equivalent:
  - (i) $X \subseteq X^*$ and X is closed.
  - (ii) X is the intersection of sets A_i maximal with respect to having $A_i \subseteq A_i^*$. [**Bergman**]
- Are there free fields? How about free characteristic zero fields? [**Rhodes**]
- What is the relation between freeness and adjoints? [**Rhodes**]

Category Theory

- Defined "representable functor" and give examples. [**Bergman**]
- What can you say about the collection of all objects X which represent a given representable functor? [**Vojta**]
- Give an example of a non-representable functor.
- Make "generators and relations" into a functor. What is its left adjoint? [**Bergman**]
- Tell us in some kind of understandable way about Quillan's Theorems A and B. [**Rhodes**]
- What do the theorems say for posets? [**Rhodes**]
- What do they say for the one object category corresponding to a group G? [**Rhodes**]
- Describe the Yoneda Embedding and prove that it is free. [**Rhodes**]
- Define monomorphism, epimorphism, injection, surjection (where appropriate). How are these connected?